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NEW YORK UNIVERSITY
Institute of Mathematical Sciences
Division of Electromagnetic Research

RESEARCH REPORT No. EM-123

Diffraction of a Plane Wave by a Right Angled Wedge Which Sustains Surface Waves on One Face

FRANK C. KARAL, JR. and SAMUEL N. KARP

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Abstract

We consider the diffraction of an electromagnetic plane wave incident on a right angled wedge with an impedance type boundary condition prescribed on one surface and a homogeneous boundary condition prescribed on the other. The impedance boundary condition is such that surface waves are generated. We present an exact mathematical solution and find the amplitude of the surface wave. A simple asymptotic representation for the far field is also given.

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1. Introduction

We consider the diffraction of an electromagnetic plane wave incident on a right angled wedge with an impedance type boundary condition prescribed on one surface and a simple boundary condition (that for perfect reflection) prescribed on the other. The impedance boundary condition is such that surface waves are generated. We present an exact mathematical solution for this problem and find the amplitude of the surface wave. We also give an asymptotic representation for the far field.

The impedance boundary conditions which give rise to surface waves and which are appropriate for a wedge of arbitrary angle, are given by

$$(1.1)\quad \begin{cases} \dfrac{1}{r}\dfrac{\partial u}{\partial \theta} + \lambda_1 u = 0 & \theta = 0 \\[2ex] \dfrac{1}{r}\dfrac{\partial u}{\partial \theta} - \lambda_2 u = 0 & \theta = \alpha \end{cases}$$

where u is the z-component of the magnetic vector, r and θ are the usual polar coordinates, α is the wedge angle and λ_1, λ_2 are constants characteristic of the wedge surfaces. The value of λ is given by

$$(1.2)\qquad \lambda = i\omega\varepsilon Z = i\omega\varepsilon(R-iX)$$

where ε is the permittivity of free space, ω is the angular frequency and Z, R, and X are the impedance, resistance and reactance of the surface, respectively. It is well known that for homogeneous media of large finite conductivity, R and X are positive in sign, small in magnitude and approximately equal. For illustrations of this boundary condition see References [3], [6], [8], and [10]. For corrugated or dielectric-coated surfaces, although R and X are positive in sign, R is much smaller than X and hence Z is almost purely reactive. See References [1], [2], [4], [5], [7] and [9]. We note that when X is positive, (1.2) implies

$$(1.3)\qquad \operatorname{Re} \lambda > 0.$$

We employ this condition in our mathematical work. The authors[13] have used this condition in studying the electromagnetic field produced by a line source located at the vertex of a right angled wedge, both of whose surfaces obey the same impedance boundary conditions.

The problem treated in this paper is not separable because of the mixed boundary condition. This difficulty can, however, be overcome by introducing an auxiliary function which is a linear combination of the magnetic field and and its cartesian derivatives. The auxiliary function is chosen in such a way that it satisfies the wave equation and simple homogeneous boundary conditions on both wedge surfaces. Such functions are easy to obtain. Once the auxiliary function is found, the original field can be determined by solving an auxiliary partial differential equation. This idea is due to Stoker[18] and Lewy[14] who studied problems in water wave theory. It has since been employed by the authors (see [11], [12] and [13]) for solving problems in diffraction theory. Once the boundary conditions have been simplified by using this idea, the development in this paper differs considerably from that occurring in water wave theory since the conditions at infinity and at the edge of the wedge are quite different. Also, the solution of the auxiliary partial differential equation involves integration in the Riemann surface of the multivalued auxiliary function. This complication is caused by condition (1.3). A more general method of water wave theory (see Peters, [16]) has been adapted by Senior[17], who solves the two-dimensional scalar problem of a plane wave incident on a wedge of completely arbitrary angle, under certain impedance boundary conditions. Surface waves do not exist in Senior's problem because of the nature of the impedance conditions employed by him.*

---

* After completing the present work, the authors were informed by Senior that he could also have treated the case of an impedance boundary condition that supports surface waves.

In Section 2 we solve the problem stated in the first paragraph. The magnitude of the surface wave propagated along the wedge surface is found for arbitrary λ. Section 3 contains the magnitude of the surface wave for the special cases of small λ and large λ. A discussion of the far field is given in Section 4.

2. Solution

Consider a right angled wedge defined by the surfaces $y = 0$, $x > 0$ and $x = 0$, $y < 0$, as shown in Figure 1. In the angular region $0 \leq \theta \leq \frac{3\pi}{2}$ we assume that we have free space. A plane wave u_I, whose magnetic vector is linearly polarized in the z-direction, is incident upon the wedge. If the angle of incidence is θ_o and the direction of the normal to the plane wave front is as shown in Figure 1, then

$$(2.1) \qquad u_I = U_o e^{-ikr\cos(\theta-\theta_o)}$$

where U_o is the magnitude of the incident plane wave and the time dependence $e^{-i\omega t}$ is omitted for convenience. The boundary conditions on the wedge surface are given by

$$(2.2) \qquad \begin{cases} \dfrac{\partial u}{\partial y} = 0 & y = 0, \quad x > 0 \\ \\ \dfrac{\partial u}{\partial x} - \lambda u = 0 & x = 0, \quad y < 0 \end{cases}$$

where u is the z-component of the magnetic vector and λ is a quantity related to the impedance. We also require that the scattered waves* at infinity be outgoing and that the energy of the electromagnetic field be finite. We wish to solve Maxwell's equations subject to the prescribed conditions and obtain the amplitude of the surface wave. We restrict θ_o in our work so that $0 \leq \theta_o < \frac{3\pi}{2}$. We leave open the possibility that for $\theta_o = \frac{3\pi}{2}$ the problem has no solution.

The time dependent form of Maxwell's equations is

$$(2.3) \qquad \begin{cases} \text{curl } \vec{H} = -i\omega\varepsilon\vec{E} \\ \\ \text{curl } \vec{E} = i\omega\mu\vec{H} \end{cases}$$

---

* By scattered waves we mean the total field minus the incident field.

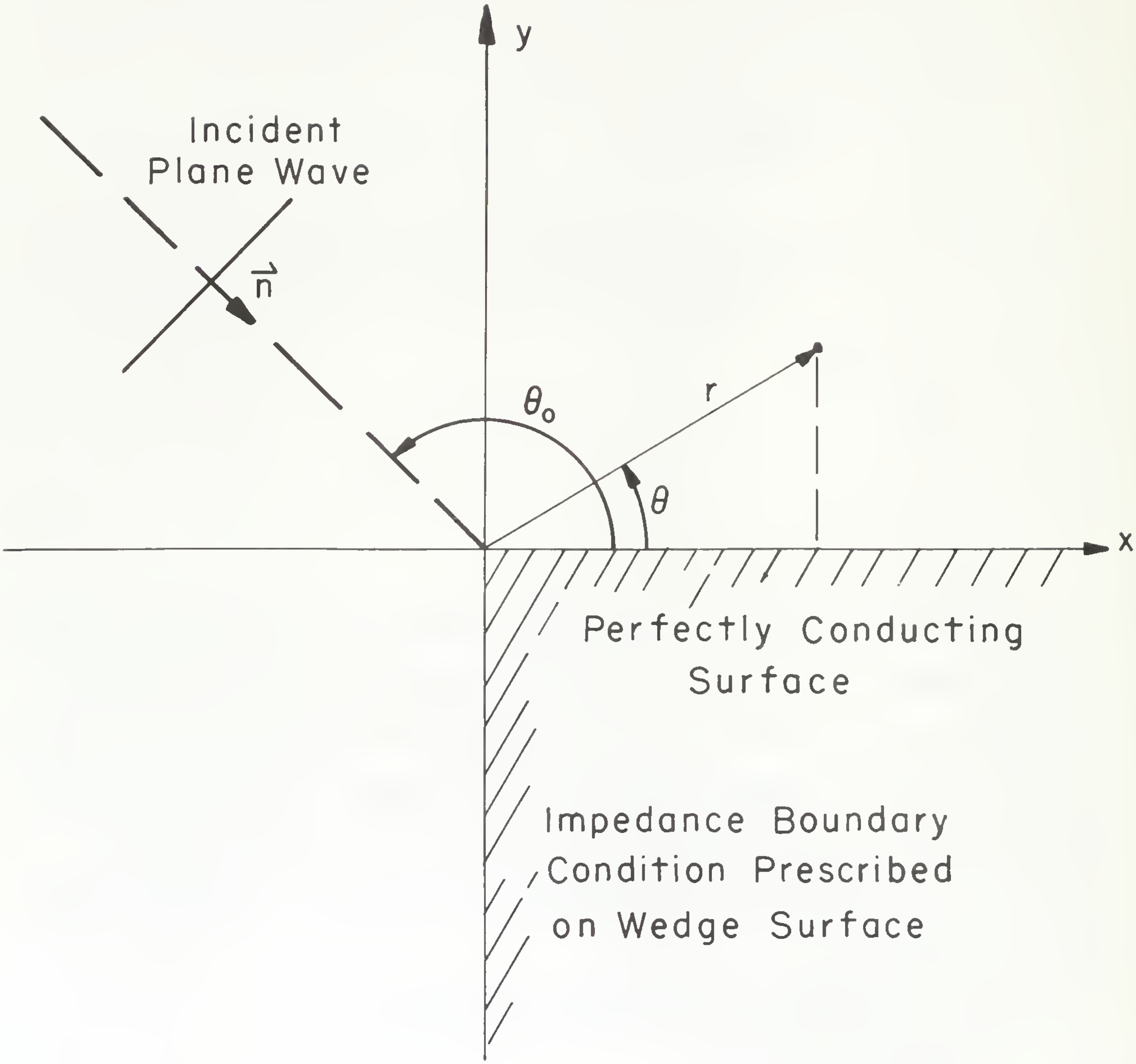

y
x
Incident
Plane Wave
$\vec{n}$
θ_o
r
θ
Perfectly Conducting
Surface
Impedance Boundary
Condition Prescribed
on Wedge Surface

Figure 1

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic field intensities, and ε and μ are the permittivity and magnetic permeability of free space. We assume the time dependence to be of the form $e^{-i\omega t}$. Because of the geometry, the field produced is independent of z and hence the field is completely determined by the value of H_z. We have

$$H_x = H_y = E_z = 0 \tag{2.4}$$

and

$$\begin{cases} E_x = -\dfrac{1}{i\omega\varepsilon}\dfrac{\partial Hz}{\partial y} \\[2ex] E_y = +\dfrac{1}{i\omega\varepsilon}\dfrac{\partial Hz}{\partial x} \end{cases} \tag{2.5}$$

The field component $H_z \equiv u$ satisfies the equation

$$(\nabla^2+k^2)u = 0 \tag{2.6}$$

where ∇^2 is the rectangular Laplacian and k is the propagation constant of free space. Therefore the mathematical problem reduces to solving the homogeneous wave equation (2.6) subject to the mixed boundary conditions (2.2) and an incident plane wave of the form given by (2.1). In addition to these requirements, we require that the far field be outgoing and that the electromagnetic field be finite everywhere.

Let us make the substitution

$$v = \left(\frac{\partial}{\partial x} - \lambda\right) u . \tag{2.7}$$

Then v satisfies the wave equation

$$(\nabla^2+k^2)v = 0 \tag{2.8}$$

subject to the simpler boundary conditions

$$\begin{cases} \dfrac{\partial v}{\partial y} = 0 & y = 0,\ x > 0 \\[2ex] v = 0 & x = 0,\ y < 0 \end{cases} \tag{2.9}$$

and the requirement that the scattered waves at infinity be outgoing. If the problem for v can be solved, then we can obtain the solution for the problem involving u since (2.7) can be integrated. A particular solution of (2.7) is

$$(2.10) \qquad u_p(x,y) = -e^{\lambda x}\int_x^{\infty} e^{-\lambda\xi} v(\xi,y)\,d\xi .$$

The form of the incident plane wave in the transformed problem is given by

$$(2.11) \qquad v_I = V_o e^{-ikr\cos(\theta-\theta_o)}$$

where

$$(2.12) \qquad V_o = -(ik\cos\theta_o+\lambda)U_o .$$

It is important to point out that the function v does not have to be finite at the origin (tip of the wedge). Thus we should first introduce all radiating solutions, no matter how singular at the origin, that satisfy the appropriate boundary conditions for v. Out of this class of solutions we should finally select only those that yield an everywhere finite value of the electromagnetic field in the original physical problem involving u. Now, if we employ the method of separation of variables and use conditions (2.9), we find that the most general representation for u is of the form

$$(2.13) \qquad v(r,\theta) = \frac{8}{3}V_o \sum_{n=0}^{\infty} J_{\frac{2n+1}{3}}(kr)\, e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}} \cos\frac{2n+1}{3}\,\theta_o \cos\frac{2n+1}{3}\,\theta$$

$$+ \sum_{n=0}^{\infty} a_n H^{(1)}_{\frac{2n+1}{3}}(kr)\cos\frac{2n+1}{3}\theta .$$

The first summation is the finite solution for an incident plane wave of the form (2.11). The second summation contains the class of admissible singular solutions. Since v cannot be too singular, the only admissible function of the singular class is the first term. Hence

$$(2.14)\quad \begin{cases} a_0 = c_1 \\ a_n \equiv 0 \qquad n \geq 1. \end{cases}$$

The constant c_1 has yet to be determined. Equation (2.13) obviously satisfies the wave equation (2.8), the boundary conditions (2.9), gives rise to an incident plane wave of the form (2.11) and yields outgoing scattered waves at infinity. If we substitute (2.13) into (2.10) we obtain for the particular solution

$$(2.15)\quad u_p = -\frac{8}{3}V_o e^{\lambda x}\int_x^\infty e^{-\lambda\xi}\sum_{n=0}^\infty J_{\frac{2n+1}{3}}(kr)e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}}\cos\frac{2n+1}{3}\theta_o\cos\frac{2n+1}{3}\theta d\xi$$

$$-c_1 e^{\lambda x}\int_x^\infty e^{-\lambda\xi}H^{(1)}_{\frac{1}{3}}(kr)\cos\frac{\theta}{3}d\xi.$$

Notice that in (2.10) and (2.15) we integrate into the wedge when y is negative. This is possible because the wave function v vanishes on the y axis and hence can be continued analytically to positive values of x. The function u_p does not satisfy all the conditions of the problem, for it is not a wave function in regions including the negative x axis, since the y derivative is discontinuous at that line. This defect can be remedied, as will be seen below, by adding to (2.15) another solution of the wave equation as follows:

$$(2.16)\quad u = -\frac{8}{3}V_o e^{\lambda x}\int_x^\infty e^{-\lambda\xi}\sum_{n=0}^\infty J_{\frac{2n+1}{3}}(kr)e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}}\cos\frac{2n+1}{3}\theta_o\cos\frac{2n+1}{3}\theta d\xi$$

$$-c_1 e^{\lambda x}\int_x^\infty e^{-\lambda\xi}H^{(1)}_{\frac{1}{3}}(kr)\cos\frac{\theta}{3}d\xi + \begin{cases} c_2 e^{\lambda x - i\sqrt{k^2+\lambda^2}\,y}, & y<0 \\ 0, & y>0\,. \end{cases}$$

The introduction of the additional term with the unknown constant c_2 now makes it possible to satisfy the continuity condition involving the y derivative of u. We note that the form of the additional term is that of a surface wave.

We must now determine the unknown constants c_1 and c_2. We do this by requiring that

$$\begin{cases} \text{(I)} & u \text{ be continuous across the line } y = 0, \quad x < 0 \\ \text{(II)} & \dfrac{\partial u}{\partial y} \text{ be continuous across the line } y = 0, \quad x < 0. \end{cases}$$

Expressing the above as jump conditions we have

$$(2.17) \quad \begin{aligned} &\text{(I)} \quad [u] = \left\{ u(x,0^+) - u(x,0^-) \right\} = 0, \qquad x < 0 \\ &\text{(II)} \quad \left[\frac{\partial u}{\partial y}\right] = \left\{ \frac{\partial u}{\partial y}(x,0^+) - \frac{\partial u}{\partial y}(x,0^-) \right\} = 0, \qquad x < 0. \end{aligned}$$

Let us now determine $\frac{\partial u}{\partial y}$. If we interchange the summation and integration signs and rearrange terms, we have

$$(2.18) \quad \begin{aligned} \frac{\partial u}{\partial y} = &- \frac{8}{3} V_o e^{-i\frac{\pi}{6}} \cos\frac{\theta_o}{3} e^{\lambda x} \int_x^\infty e^{-\lambda\xi} \frac{\partial}{\partial y} \left\{ J_{\frac{1}{3}}(kr) \cos\frac{\theta}{3} \right\} d\xi \\ &- \frac{8}{3} V_o e^{\lambda x} \sum_{n=0}^\infty e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}} \cos\left(\frac{2n}{3}+1\right)\theta_o \int_x^\infty e^{-\lambda\xi} \frac{\partial}{\partial y} \left\{ J_{\frac{2n}{3}+1}(kr) \cos\left(\frac{2n}{3}+1\right)\theta \right\} d\xi \\ &- c_1 e^{\lambda x} \int_x^\infty e^{-\lambda\xi} \frac{\partial}{\partial y} \left\{ H^{(1)}_{\frac{1}{3}}(kr) \cos\frac{\theta}{3} \right\} d\xi + \begin{cases} -i\sqrt{k^2+\lambda^2}\, c_2 e^{\lambda x - i\sqrt{k^2+\lambda^2}\, y}, & y<0 \\ 0 & , y>0. \end{cases} \end{aligned}$$

Now

$$(2.19) \begin{cases} \dfrac{\partial}{\partial y}\left\{ H^{(1)}_{\frac{1}{3}}(kr)\cos\dfrac{\theta}{3} \right\} = \dfrac{\partial}{\partial x}\left\{ H^{(1)}_{\frac{1}{3}}(kr)\sin\dfrac{\theta}{3} \right\} + k e^{i\frac{2\pi}{3}} H^{(1)}_{\frac{2}{3}}(kr)\sin\dfrac{2\theta}{3} \\ \dfrac{\partial}{\partial y}\left\{ J_{\frac{2n}{3}+1}(kr)\cos\left(\dfrac{2n}{3}+1\right)\theta \right\} = \dfrac{\partial}{\partial x}\left\{ J_{\frac{2n}{3}+1}(kr)\sin\left(\dfrac{2n}{3}+1\right)\theta \right\} - kJ_{\frac{2n}{3}}(kr)\sin\dfrac{2n}{3}\theta \\ \dfrac{\partial}{\partial y}\left\{ J_{\frac{1}{3}}(kr)\cos\dfrac{\theta}{3} \right\} = \dfrac{\partial}{\partial x}\left\{ J_{\frac{1}{3}}(kr)\sin\dfrac{\theta}{3} \right\} + kJ_{-\frac{2}{3}}(kr)\sin\dfrac{2\theta}{3}. \end{cases}$$

Substituting (2.19) into (2.18) and integrating by parts we have

$$(2.20)\quad \frac{\partial u}{\partial y} = +\frac{8}{3}V_o e^{-i\frac{\pi}{6}}\cos\frac{\theta_o}{3}\left\{J_{\frac{1}{3}}(kr)\sin\frac{\theta}{3}\right\} - \frac{8}{3}V_o\lambda e^{-i\frac{\pi}{6}}\cos\frac{\theta_o}{3}e^{\lambda x}\int_x^\infty e^{-\lambda\xi}J_{\frac{1}{3}}(kr)\sin\frac{\theta}{3}d\xi$$

$$-\frac{8}{3}V_o k e^{-i\frac{\pi}{6}}\cos\frac{\theta_o}{3}e^{\lambda x}\int_x^\infty e^{-\lambda\xi}J_{-\frac{2}{3}}(kr)\sin\frac{2\theta}{3}d\xi$$

$$+\frac{8}{3}V_o\sum_{n=0}^{\infty}e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}}\cos\left(\frac{2n}{3}+1\right)\theta_o J_{\frac{2n}{3}+1}(kr)\sin\left(\frac{2n}{3}+1\right)\theta$$

$$-\frac{8}{3}V_o\lambda e^{\lambda x}\sum_{n=0}^{\infty}e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}}\cos\left(\frac{2n}{3}+1\right)\theta_o\int_x^\infty e^{-\lambda\xi}J_{\frac{2n}{3}+1}(kr)\sin\left(\frac{2n}{3}+1\right)\theta d\xi$$

$$+\frac{8}{3}V_o k e^{\lambda x}\sum_{n=0}^{\infty}e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}}\cos\left(\frac{2n}{3}+1\right)\theta_o\int_x^\infty e^{-\lambda\xi}J_{\frac{2n}{3}}(kr)\sin\frac{2n}{3}\theta d\xi$$

$$+c_1H^{(1)}_{\frac{1}{3}}(kr)\sin\frac{\theta}{3} - c_1\lambda e^{\lambda x}\int_x^\infty e^{-\lambda\xi}H^{(1)}_{\frac{1}{3}}(kr)\sin\frac{\theta}{3}d\xi$$

$$-c_1ke^{i\frac{2\pi}{3}}e^{\lambda x}\int_x^\infty e^{-\lambda\xi}H^{(1)}_{\frac{2}{3}}(kr)\sin\frac{2\theta}{3}d\xi + \begin{cases}-i\sqrt{k^2+\lambda^2}\;c_2e^{\lambda x-i\sqrt{k^2+\lambda^2}y} & y<0\\ 0 & ,\ y>0.\end{cases}$$

Let us now impose jump condition (I). We have

$$(2.21)\quad +\frac{3}{2}I_{\frac{1}{3}}c_1+c_2 = +\frac{8}{3}V_o\sum_{n=0}^{\infty}L_{\frac{2n+1}{3}}e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}}\cos\frac{2n+1}{3}\theta\,\cos\frac{2n+1}{3}2\pi$$

$$-\frac{8}{3}V_o\sum_{n=0}^{\infty}L_{\frac{2n+1}{3}}e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}}\cos\frac{2n+1}{3}\theta_o$$

where

$$(2.22)\quad I_\nu = \int_0^\infty e^{-\lambda\xi} H_\nu^{(1)}(k|\xi|)\,d\xi$$

$$= \frac{\left(\sqrt{\lambda^2+k^2} - \lambda\right)^\nu}{k^\nu \sqrt{\lambda^2+k^2}} \left\{ 1 + \frac{i}{\sin\nu\pi} \left[\cos\nu\pi - \frac{\left(\lambda + \sqrt{\lambda^2+k^2}\right)^{2\nu}}{k^{2\nu}} \right] \right\}$$

and

$$(2.23)\quad L_\nu = \int_0^\infty e^{-\lambda\xi} J_\nu(k|\xi|)\,d\xi$$

$$= \frac{1}{\sqrt{\lambda^2 = k^2}} \; \frac{k^\nu}{\left(\lambda + \sqrt{\lambda^2+k^2}\right)^\nu} \; .$$

(See Magnus and Oberhettinger[15].) We next impose jump condition (II). We obtain

$$(2.24)\quad \frac{\sqrt{3}}{2} \left\{ \lambda I_{\frac{1}{3}} - k e^{i\frac{2\pi}{3}} I_{\frac{2}{3}} \right\} c_1 + i\sqrt{k^2+\lambda^2}\, c_2$$

$$= -\frac{4\sqrt{3}}{3} V_o \lambda e^{-i\frac{\pi}{6}} L_{\frac{1}{3}} \cos\frac{\theta_o}{3} + \frac{4\sqrt{3}}{3} V_o k e^{-i\frac{\pi}{6}} L_{-\frac{2}{3}} \cos\frac{\theta_o}{3}$$

$$- \frac{8}{3} V_o \lambda \sum_{n=0}^{\infty} L_{\frac{2n}{3}+1}\, e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}} \cos\left(\frac{2n}{3}+1\right)\theta_o \sin\left(\frac{2n}{3}+1\right) 2\pi$$

$$+ \frac{8}{3} V_o k \sum_{n=0}^{\infty} L_{\frac{2n}{3}}\, e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}} \cos\left(\frac{2n}{3}+1\right)\theta_o \, \sin\frac{4\pi n}{3} \; .$$

If we let

$$(2.25)\qquad A = \frac{k}{\lambda + \sqrt{k^2+\lambda^2}}$$

then

$$(2.26)\qquad L_\nu = \frac{1}{\sqrt{k^2+\lambda^2}} A^\nu$$

and (2.21) and (2.24) become

$$(2.27)\quad \frac{3}{2} I_{\frac{1}{3}} c_1 + c_2 = \frac{8}{3} V_o \sum_{n=0}^{\infty} \frac{A^{\frac{2n+1}{3}}}{\sqrt{k^2+\lambda^2}} e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}} \cos\frac{2n+1}{3}\,\theta_o \cos\frac{2n+1}{3}\,2\pi$$

$$- \frac{8}{3} V_o \sum_{n=0}^{\infty} \frac{A^{\frac{2n+1}{3}}}{\sqrt{k^2+\lambda^2}} e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}} \cos\frac{2n+1}{3}\,\theta_o$$

and

$$(2.28)\quad \frac{\sqrt{3}}{2}\left\{\lambda I_{\frac{1}{3}} - k e^{i\frac{2\pi}{3}} I_{\frac{2}{3}}\right\} c_1 + i\sqrt{k^2+\lambda^2}\, c_2$$

$$= -\frac{4\sqrt{3}}{3} V_o \frac{\lambda}{\sqrt{k^2+\lambda^2}} A^{\frac{1}{3}} e^{-i\frac{\pi}{6}} \cos\frac{\theta_o}{3} + \frac{4\sqrt{3}}{3} V_o \frac{k}{\sqrt{k^2+\lambda^2}} A^{-\frac{2}{3}} e^{-i\frac{\pi}{6}} \cos\frac{\theta_o}{3}$$

$$- \frac{8}{3} V_o \lambda \sum_{n=0}^{\infty} \frac{1}{\sqrt{k^2+\lambda^2}} A^{\frac{2n}{3}+1} e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}} \cos\left(\frac{2n}{3}+1\right)\theta_o \sin\left(\frac{2n}{3}+1\right)2\pi$$

$$+ \frac{8}{3} V_o k \sum_{n=0}^{\infty} \frac{1}{\sqrt{k^2+\lambda^2}} A^{\frac{2n}{3}} e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}} \cos\left(\frac{2n}{3}+1\right)\theta_o \sin\frac{4\pi n}{3}\,.$$

The summations appearing in (2.27) and (2.28) can be evaluated by substituting exponentials for the trigonometrical functions and summing the resulting geometrical series. It can be shown that

$$(2.29)\quad \sum_{n=0}^{\infty} A^{\frac{2n+1}{3}} e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}} \cos\frac{2n+1}{3}\,\theta_o \cos\frac{2n+1}{3}\,2\pi = \frac{A^{1/3}}{4}\delta_1$$

$$(2.30)\quad \sum_{n=0}^{\infty} A^{\frac{2n+1}{3}} e^{-i\left(\frac{2n+1}{3}\right)\frac{\pi}{2}} \cos\frac{2n+1}{3}\,\theta_o = \frac{A^{1/3}}{2}\delta_2$$

$$(2.31)\quad \sum_{n=0}^{\infty} A^{\frac{2n}{3}+1} e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}} \cos\left(\frac{2n}{3}+1\right)\theta_o \sin\left(\frac{2n}{3}+1\right)2\pi = -\frac{A}{4}\delta_4$$

$$(2.32)\qquad \sum_{n=0}^{\infty} A^{\frac{2n}{3}} e^{-i\left(\frac{2n}{3}+1\right)\frac{\pi}{2}} \cos\left(\frac{2n}{3}+1\right)\theta_o \sin\frac{4\pi n}{3} = -\frac{1}{4}\mathcal{S}_4$$

where

$$(2.33)\qquad \mathcal{S}_1 = \left[\frac{\exp\left[i\frac{1}{3}\left(\theta_o+\frac{3\pi}{2}\right)\right]}{1-A^{2/3}\exp\left[i\frac{2}{3}\left(\theta_o+\frac{3\pi}{2}\right)\right]} + \frac{\exp\left[i\frac{1}{3}\left(\theta_o-\frac{5\pi}{2}\right)\right]}{1-A^{2/3}\exp\left[i\frac{2}{3}\left(\theta_o-\frac{5\pi}{2}\right)\right]}\right.$$

$$\left. + \frac{\exp\left[-i\frac{1}{3}\left(\theta_o-\frac{3\pi}{2}\right)\right]}{1-A^{2/3}\exp\left[-i\frac{2}{3}\left(\theta_o-\frac{3\pi}{2}\right)\right]} + \frac{\exp\left[-i\frac{1}{3}\left(\theta_o+\frac{5\pi}{2}\right)\right]}{1-A^{2/3}\exp\left[-i\frac{2}{3}\left(\theta_o+\frac{5\pi}{2}\right)\right]}\right]$$

$$(2.34)\qquad \mathcal{S}_2 = \left[\frac{\exp\left[i\frac{1}{3}\left(\theta_o-\frac{\pi}{2}\right)\right]}{1-A^{2/3}\exp\left[i\frac{2}{3}\left(\theta_o-\frac{\pi}{2}\right)\right]} + \frac{\exp\left[-i\frac{1}{3}\left(\theta_o+\frac{\pi}{2}\right)\right]}{1-A^{2/3}\exp\left[-i\frac{2}{3}\left(\theta_o+\frac{\pi}{2}\right)\right]}\right]$$

$$(2.35)\qquad \mathcal{S}_4 = \left[\frac{e^{i\theta_o}}{1-A^{2/3}\exp\left[i\frac{2}{3}\left(\theta_o+\frac{3\pi}{2}\right)\right]} - \frac{e^{i\theta_o}}{1-A^{2/3}\exp\left[i\frac{2}{3}\left(\theta_o+\frac{5\pi}{2}\right)\right]}\right.$$

$$\left. + \frac{e^{-i\theta_o}}{1-A^{2/3}\exp\left[-i\frac{2}{3}\left(\theta_o-\frac{3\pi}{2}\right)\right]} - \frac{e^{-i\theta_o}}{1-A^{2/3}\exp\left[-i\frac{2}{3}\left(\theta_o+\frac{5\pi}{2}\right)\right]}\right].$$

The above formulas can be written more compactly by making the substitution

$$(2.36)\qquad A = \frac{k}{\lambda+\sqrt{\lambda^2+k^2}} = e^{i\Psi}$$

and combining terms. We obtain

$$(2.37)\qquad \mathcal{S}_1 = 2ie^{-i\frac{1}{3}\Psi}\left[\frac{\sin\frac{1}{3}\left(\Psi+\frac{3\pi}{2}\right)\cos\frac{1}{3}\theta_o}{\cos\frac{2}{3}\theta_o-\cos\frac{2}{3}\left(\Psi+\frac{3\pi}{2}\right)} + \frac{\sin\frac{1}{3}\left(\Psi-\frac{5\pi}{2}\right)\cos\frac{1}{3}\theta_o}{\cos\frac{2}{3}\theta_o-\cos\frac{2}{3}\left(\Psi-\frac{5\pi}{2}\right)}\right]$$

$$(2.38)\qquad \mathcal{S}_2 = 2ie^{-i\frac{1}{3}\Psi}\left[\frac{\sin\frac{1}{3}\left(\Psi-\frac{\pi}{2}\right)\cos\frac{1}{3}\theta_o}{\cos\frac{2}{3}\theta_o-\cos\frac{2}{3}\left(\Psi-\frac{\pi}{2}\right)}\right]$$

$$(2.39)\qquad \mathcal{S}_4 = \left[\frac{\cos\frac{1}{3}\theta_o-\exp\left[-i\frac{2}{3}\left(\Psi+\frac{3\pi}{2}\right)\right]\cos\theta_o}{\cos\frac{2}{3}\theta_o-\cos\frac{2}{3}\left(\Psi+\frac{3\pi}{2}\right)} - \frac{\cos\frac{1}{3}\theta_o-\exp\left[-i\frac{2}{3}\left(\Psi-\frac{5\pi}{2}\right)\right]\cos\theta_o}{\cos\frac{2}{3}\theta_o-\cos\frac{2}{3}\left(\Psi-\frac{5\pi}{2}\right)}\right].$$

Note that Ψ is a complex angle. The formulas for I_ν and L_ν in terms of this notation become

$$(2.40) \qquad I_\nu = \frac{-2}{\sqrt{k^2+\lambda^2}} \frac{e^{-i\nu\frac{\pi}{2}}}{\sin\nu\pi} \sin\nu\left(\Psi - \frac{\pi}{2}\right)$$

and

$$(2.41) \qquad L_\nu = \frac{1}{\sqrt{k^2+\lambda^2}} e^{i\nu\Psi} .$$

By making the substitutions given by (2.33), (2.34) and (2.35), equations (2.27) and (2.28) become

$$(2.42) \qquad \frac{3}{2} I_{\frac{1}{3}} c_1 + c_2 = \frac{2V_o A^{1/3}}{3\sqrt{k^2+\lambda^2}} \left\{ \mathcal{S}_1 - 2\mathcal{S}_2 \right\}$$

and

$$(2.43) \qquad \frac{\sqrt{3}}{2} \left\{ \lambda I_{\frac{1}{3}} - k e^{i\frac{2\pi}{3}} I_{\frac{2}{3}} \right\} c_1 + i\sqrt{k^2+\lambda^2}\, c_2$$

$$= \frac{2V_o}{3\sqrt{k^2+\lambda^2}} \left\{ \mathcal{S}_4 - 2\sqrt{3}\, e^{-i\frac{\pi}{6}} A^{-\frac{2}{3}} \cos\frac{\theta_o}{3} \right\} (\lambda A - k) \quad .$$

Solving for c_1 and c_2 we obtain

$$(2.44) \qquad \triangle \cdot c_1 = \frac{2V_o i A^{1/3}}{3} \left\{ \mathcal{S}_1 - 2\mathcal{S}_2 \right\} - \frac{2V_o}{3\sqrt{k^2+\lambda^2}} (\lambda A - k) \left\{ \mathcal{S}_4 - 2\sqrt{3}\, e^{-i\frac{\pi}{6}} A^{-\frac{2}{3}} \cos\frac{\theta_o}{3} \right\}$$

and

$$(2.45) \qquad \triangle \cdot c_2 = -\frac{\sqrt{3}\, V_o A^{1/3}}{3\sqrt{k^2+\lambda^2}} \left\{ \mathcal{S}_1 - 2\mathcal{S}_2 \right\} \left\{ \lambda I_{\frac{1}{3}} - k e^{i\frac{2\pi}{3}} I_{\frac{2}{3}} \right\}$$

$$+ \frac{V_o I_{1/3}}{\sqrt{k^2+\lambda^2}} (\lambda A - k) \left\{ \mathcal{S}_4 - 2\sqrt{3}\, e^{-i\frac{\pi}{6}} A^{-\frac{2}{3}} \cos\frac{\theta_o}{3} \right\}$$

where

$$(2.46)\quad \Delta = \frac{3}{2} i \sqrt{k^2+\lambda^2}\, I_{\frac{1}{3}} - \frac{\sqrt{3}}{2} \left\{ \lambda I_{\frac{1}{3}} - k e^{i\frac{2\pi}{3}} I_{\frac{2}{3}} \right\} .$$

The amplitude of the surface wave is given by c_2.

3. Limiting cases for the amplitudes of the surface waves

We first consider the case of small λ. It is easily shown that

$$(3.1)\qquad A \to I$$

$$(3.2)\qquad I_{\frac{1}{3}} \to \frac{1}{k}\left(1-\frac{i}{\sqrt{3}}\right)$$

$$(3.3)\qquad I_{\frac{2}{3}} \to \frac{1}{k}(1-i\sqrt{3})$$

$$(3.4)\qquad \mathcal{S}_1 \to 2i\left[\frac{\cos\frac{1}{3}\theta_o}{\cos\frac{2}{3}\theta_o+1} - \frac{\cos\frac{1}{3}\theta_o}{2\cos\frac{2}{3}\theta_o-1}\right] \equiv \hat{\mathcal{S}}_1$$

$$(3.5)\qquad \mathcal{S}_2 \to -2i\left[\frac{\cos\frac{1}{3}\theta_o}{2\cos\frac{2}{3}\theta_o-1}\right] \equiv \hat{\mathcal{S}}_2$$

$$(3.6)\qquad \mathcal{S}_4 \to \left[\frac{\cos\theta_o+\cos\frac{1}{3}\theta_o}{\cos\frac{2}{3}\theta_o+1} + 2\frac{e^{i\frac{5\pi}{3}}\cos\theta_o-\cos\frac{1}{3}\theta_o}{2\cos\frac{2}{3}\theta_o-1}\right] \equiv \hat{\mathcal{S}}_4$$

$$(3.7)\qquad \triangle \to \frac{3}{2}(\sqrt{3}+i)\ .$$

If we rearrange (2.45) in the following form

$$(3.8)\qquad \triangle c_2 = \frac{\sqrt{3}V_o A^{1/3} I_{1/3}\lambda}{3\sqrt{k^2+\lambda^2}}\left\{\sqrt{3}\,A^{2/3}\mathcal{S}_4 - 6e^{-i\frac{\pi}{6}}\cos\frac{\theta_o}{3} - \mathcal{S}_1+2\mathcal{S}_2\right\}$$

$$+\frac{\sqrt{3}\,V_o k}{3\sqrt{k^2+\lambda^2}}\left\{-\sqrt{3}\,I_{\frac{1}{3}}\mathcal{S}_4+6e^{-i\frac{\pi}{6}}I_{\frac{1}{3}}A^{-\frac{2}{3}}\cos\frac{\theta_o}{3}\right.$$

$$\left.+e^{i\frac{2\pi}{3}}A^{1/3}I_{\frac{2}{3}}\left[\mathcal{S}_1-2\mathcal{S}_2\right]\right\},$$

and use the approximations already given, it can be shown that the second group of terms is zero. Hence, for small λ,

$$(3.9)\qquad \triangle c_2 \to \frac{V_o\lambda}{3k^2}(\sqrt{3}-i)\left\{\sqrt{3}\hat{\mathcal{S}}_4-3(\sqrt{3}-i)\cos\frac{\theta_o}{3} - \hat{\mathcal{S}}_1+2\hat{\mathcal{S}}_2\right\},$$

or, after some manipulation,

$$(3.10)\qquad c_2 \rightarrow +\frac{4}{3}U_o e^{i\frac{\pi}{3}}\left(\frac{\lambda}{k}\right)\cos\frac{2}{3}\theta_o .$$

We note that the amplitude of the surface wave approaches zero as λ approaches zero.

We next consider the case of large λ. It is easily shown that

$$(3.11)\qquad A \rightarrow \frac{k}{2\lambda}$$

$$(3.12)\qquad I_{\frac{1}{3}} \rightarrow -i\,\frac{2}{\sqrt{3}}\left(\frac{2}{k}\right)^{2/3}\frac{1}{\lambda^{2/3}}$$

$$(3.13)\qquad I_{\frac{2}{3}} \rightarrow -i\,\frac{2}{\sqrt{3}}\left(\frac{2}{k}\right)^{2/3}\frac{1}{\lambda^{1/3}}$$

$$(3.14)\qquad \mathcal{S}_1 \rightarrow 2\cos\frac{1}{3}\,\theta_o\left[i+e^{-i\frac{5\pi}{6}}\right]$$

$$(3.15)\qquad \mathcal{S}_2 \rightarrow 2e^{-i\frac{\pi}{6}}\cos\frac{1}{3}\theta_o$$

$$(3.16)\qquad \mathcal{S}_4 \rightarrow 0$$

$$(3.17)\qquad \triangle \rightarrow 2e^{+i\frac{\pi}{6}}\left(\frac{2}{k}\right)^{2/3}\lambda^{1/3} .$$

If we make use of the above approximations and substitute into (2.45) we obtain

$$(3.18)\qquad c_2 \rightarrow -\frac{e^{i\frac{\pi}{6}}2V_o k}{\lambda^2}\left(\frac{2\lambda}{k}\right)^{2/3}\cos\frac{1}{3}\theta_o$$

or

$$(3.19)\qquad c_2 \rightarrow +U_o e^{i\frac{\pi}{6}}\left(\frac{2k}{\lambda}\right)^{1/3}\cos\frac{1}{3}\theta_o .$$

We note that the amplitude of the surface wave approaches zero as λ approaches infinity. Since the amplitude approaches zero for both small and large λ, there is some optimum value of λ for which the magnitude of the surface wave is a maximum. This optimum value may depend on the angle of incidence θ_o.

4. Far field evaluation of the electromagnetic field

We have already obtained the exact solution of the diffraction problem. However, it is desirable to find a simple expression for the far field. Now

$$(4.1) \qquad v = \left(\frac{\partial}{\partial x} - \lambda\right)u.$$

The original electric field component u in the far field can be written as the sum of two terms. The first term can be found by using geometrical optics. The second is due to the diffracted field. We have

$$(4.2) \qquad u = (u)_{geom} + (u)_{diff}$$

where $(u)_{geom}$ is assumed to be known, and $(u)_{diff}$ is to be found. The transformed electric field component v in the far field can be written in the form

$$(4.3) \qquad v = (v)_{geom} + \left[(v)_{diff}\right]_F + \left[(v)_{diff}\right]_S .$$

The far field solution of the transformed problem which is finite at the edge of the wedge, and which is denoted by the subscript F in (4.3), can be obtained from Reiche[19]. Reiche uses the method of reflection on Riemann sheets due to Sommerfeld[20] and applies it to the case of a plane wave diffracted by a right angled wedge where the boundary conditions require the vanishing of the field on the surface. By using some of Reiche's intermediate results, it is easy to solve the problem of the diffraction of a plane wave by a right angled wedge where the boundary conditions require the vanishing of the field on one surface $\left(\theta = \frac{3\pi}{2}\right)$ and the vanishing of the derivative on the other $(\theta = 0)$. We note that the only required changes in notation between the results of Reiche and those used here are the following: (1) replace i by -i since we use the time factor $e^{-i\omega t}$; and (2) multiply the solution by V_0 in order to have the proper amplitude. The far field solution of the transformed problem which is singular at the edge of the wedge, and which is denoted by the subscript S in (4.3), is the asymptotic form of

$$(4.4) \qquad v = c_1 H^{(1)}_{\frac{1}{3}}(kr)\cos\frac{\theta}{3}\,.$$

The constant c_1 is the same constant that appears in the exact solution. The far field expressions for the finite and singular solutions are given by

$$(4.5) \qquad \Big[(v)_{\text{diff}}\Big]_F = -\frac{2V_o e^{i\frac{\pi}{4}}}{\sqrt{6\pi kr}}\left[\frac{\sin\frac{2}{3}(\theta_o-\theta)}{\sin(\theta_o-\theta)} + \frac{\sin\frac{2}{3}(\theta_o+\theta)}{\sin(\theta_o+\theta)}\right] e^{ikr}$$

$$(4.6) \qquad \Big[(v)_{\text{diff}}\Big]_S = c_1\sqrt{\frac{2}{\pi kr}}\, e^{-i\frac{5\pi}{12}} \cos\frac{1}{3}\theta\, e^{ikr}$$

where

$$(4.7) \qquad V_o = -\left(+ik\cos\theta_o+\lambda\right) U_o.$$

We expect the far field form of the original field u to have forms similar to that given by (4.5). Therefore we write

$$(4.8) \qquad (u)_{\text{diff}} = \frac{m(\theta)}{\sqrt{r}}\, e^{ikr}$$

where $m(\theta)$ is unknown. Consider now only the diffracted part of the far field expressions. The transformation equation (4.1) becomes

$$(4.9) \qquad (v)_{\text{diff}} = \left(\frac{\partial}{\partial x} - \lambda\right)\left\{(u)_{\text{diff}}\right\}$$

or

$$(4.10) \qquad \Big[(v)_{\text{diff}}\Big]_F + \Big[(v)_{\text{diff}}\Big]_S = \left(\cos\theta\,\frac{\partial}{\partial r} - \lambda\right)(u)_{\text{diff}}$$

where we have used the far field result

$$(4.11) \qquad \frac{\partial}{\partial x} = \cos\theta\,\frac{\partial}{\partial r} - \frac{1}{r}\sin\theta\,\frac{\partial}{\partial\theta} \to \cos\theta\,\frac{\partial}{\partial r}\,.$$

Substituting (4.5), (4.6) and (4.8) into (4.10) we have

$$(4.12) \qquad -\frac{2V_o e^{i\frac{\pi}{4}}}{\sqrt{6\pi k}}\left[\frac{\sin\frac{2}{3}(\theta_o-\theta)}{\sin(\theta_o-\theta)} + \frac{\sin\frac{2}{3}(\theta_o+\theta)}{\sin(\theta_o+\theta)}\right] + c_1\sqrt{\frac{2}{\pi k}}\, e^{-i\frac{5\pi}{12}}\cos\frac{1}{3}\theta$$

$$= (ik\cos\theta - \lambda)m(\theta)$$

or

$$(4.13)\quad m(\theta) = \frac{2V_o e^{i\frac{\pi}{4}}}{\sqrt{6\pi k}\,(ik\cos\theta-\lambda)} \left\{ -\left[\frac{\sin\frac{2}{3}(\theta_o-\theta)}{\sin(\theta_o-\theta)} + \frac{\sin\frac{2}{3}(\theta_o+\theta)}{\sin(\theta_o+\theta)} \right] + \frac{\sqrt{3}\,c_1}{V_o}\, e^{-i\frac{2\pi}{3}} \cos\frac{1}{3}\theta \right\}$$

or

$$(4.14)\quad m(\theta) = \sqrt{\frac{2}{3\pi k}}\; \frac{-U_o e^{i\frac{\pi}{4}}}{(ik\cos\theta_o+\lambda)(ik\cos\theta-\lambda)} \cdot \left\{ -\left[\frac{\sin\frac{2}{3}(\theta_o-\theta)}{\sin(\theta_o-\theta)} + \frac{\sin\frac{2}{3}(\theta_o+\theta)}{\sin(\theta_o+\theta)} \right] + \left(\frac{\sqrt{3}\,c_1}{V_o}\right) e^{-i\frac{2\pi}{3}} \cos\frac{1}{3}\theta \right\}$$

where

$$(4.15)\quad \left(\frac{\sqrt{3}\,c_1}{V_o}\right) = \frac{2iA^{1/3}}{\sqrt{3}\,\triangle} \left\{ \mathcal{S}_1 - 2\mathcal{S}_2 \right\} - \frac{2}{\sqrt{3}\,\triangle\sqrt{k^2+\lambda^2}} \left\{ \mathcal{S}_4 - 2\sqrt{3}\, e^{-i\frac{\pi}{6}} A^{-\frac{2}{3}} \cos\frac{\theta_o}{3} \right\} \left\{ \lambda A - k \right\}$$

and

$$(4.16)\quad \triangle = \frac{3}{2} i \sqrt{k^2+\lambda^2}\; I_{\frac{1}{3}} - \frac{\sqrt{3}}{2} \left\{ \lambda I_{\frac{1}{3}} - k e^{i\frac{2\pi}{3}} I_{\frac{2}{3}} \right\} .$$

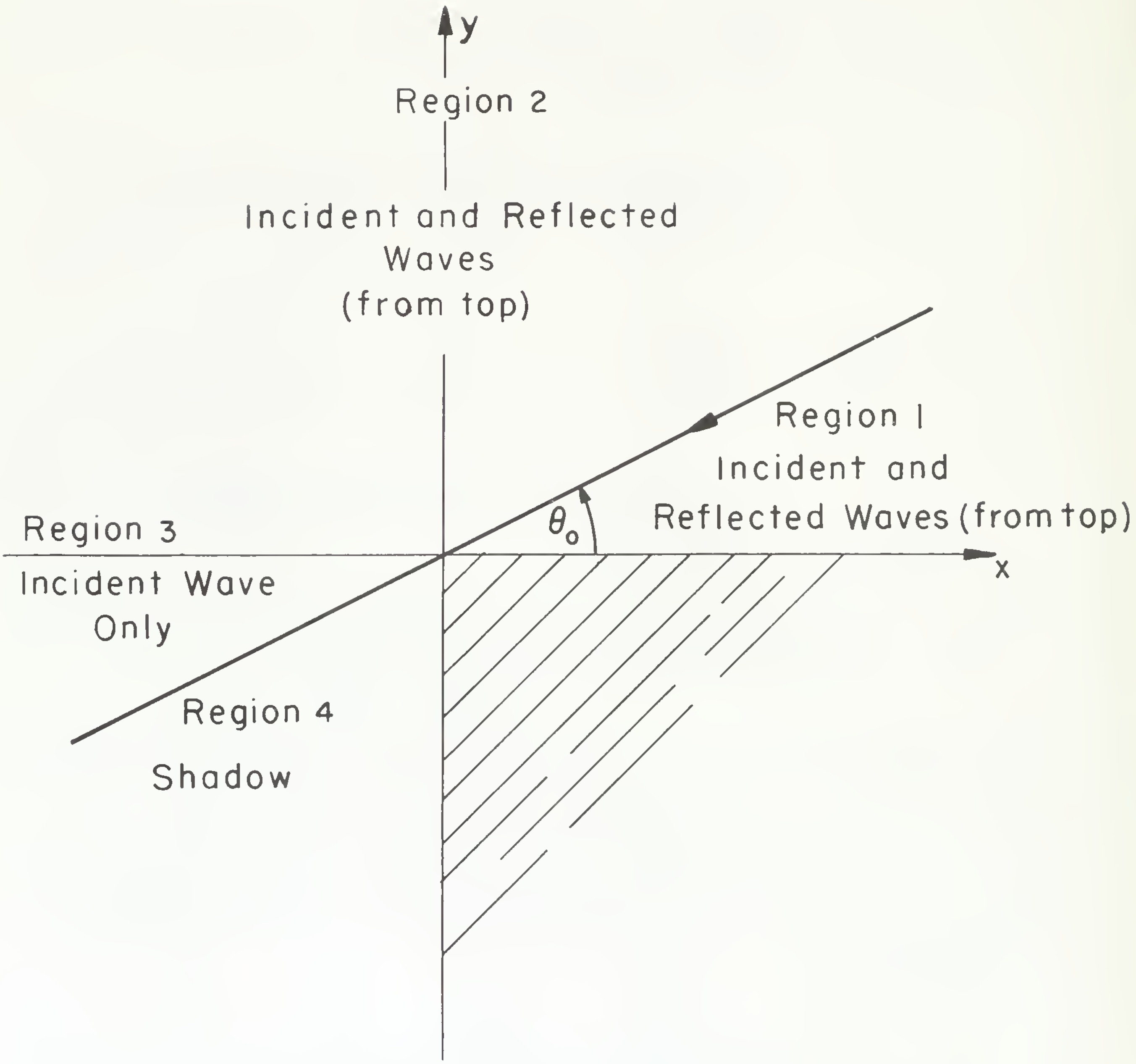
The result given by (4.14) is the desired diffracted angular dependence for the far field. The diffracted far field is given by (4.8).

We shall now summarize the results of the far field. See Figures 2, 3, and 4 for definitions of the regions referred to in Cases A, B, and C respectively.

Case A: $0 < \theta_o < \frac{\pi}{2}$

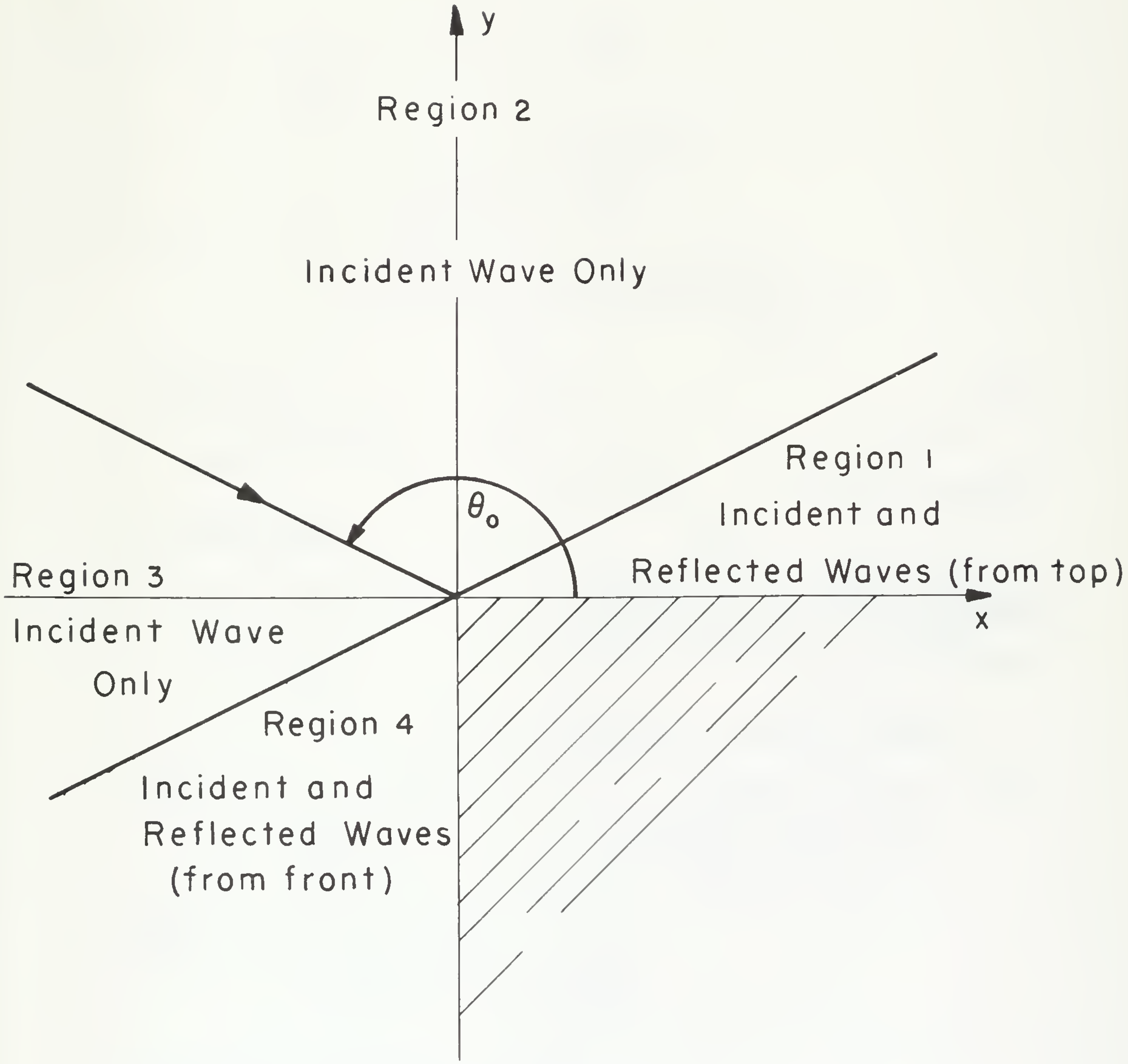
1. Regions 1 and 2: $(0 < \theta < \pi-\theta_o)$

$$u = U_o e^{-ikr\cos(\theta-\theta_o)} + U_o e^{-ikr\cos(\theta+\theta_o)} + \frac{m(\theta)}{\sqrt{r}}\, e^{ikr}$$


y
Region 2
Incident and Reflected Waves (from top)
Region 1
Incident and Reflected Waves (from top)
θ_o
Region 3
Incident Wave Only
x
Region 4
Shadow

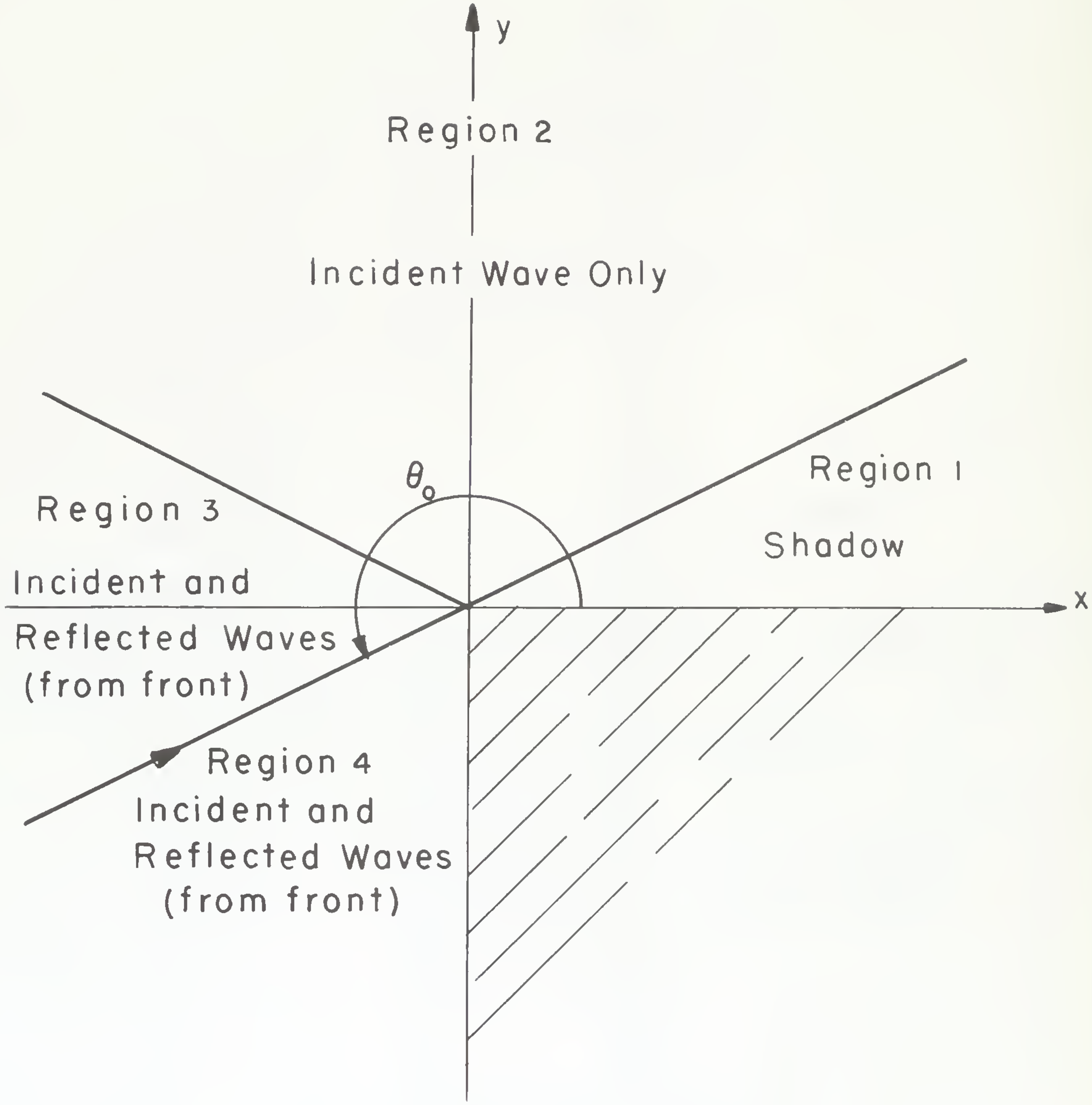
Case A: $0 < \theta_o < \frac{\pi}{2}$

Figure 2



Case B: $\frac{\pi}{2} < \theta_0 < \pi$

Figure 3



Case C: $\pi < \theta_0 < \frac{3\pi}{2}$

Figure 4

2. Region 3: $\left(\pi-\theta_o < \theta < \pi+\theta_o\right)$

$$u = U_o e^{-ikr\cos\left(\theta-\theta_o\right)} + \frac{m(\theta)}{\sqrt{r}} e^{ikr}$$

3. Region 4: $\left(\pi+\theta_o < \theta < \frac{3\pi}{2}\right)$

$$u = \frac{m(\theta)}{\sqrt{r}} e^{ikr}$$

Case B: $\frac{\pi}{2} < \theta_o < \pi$

1. Region 1: $\left(0 < \theta < \pi-\theta_o\right)$

$$u = U_o e^{-ikr\cos\left(\theta-\theta_o\right)} + U_o e^{-ikr\cos\left(\theta+\theta_o\right)} + \frac{m(\theta)}{\sqrt{r}} e^{ikr}$$

2. Regions 2 and 3: $\left(\pi-\theta_o < \theta < 2\pi-\theta_o\right)$

$$u = U_o e^{-ikr\cos\left(\theta-\theta_o\right)} + \frac{m(\theta)}{\sqrt{r}} e^{ikr}$$

3. Region 4: $\left(2\pi-\theta_o < \theta < \frac{3\pi}{2}\right)$

$$u = U_o e^{-ikr\cos\left(\theta-\theta_o\right)} + U_o \frac{ik\cos\theta_o+\lambda}{ik\cos\theta_o-\lambda} e^{+ikr\cos\left(\theta+\theta_o\right)} + \frac{m(\theta)}{\sqrt{r}} e^{ikr}$$

Case C: $\pi < \theta_o < \frac{3\pi}{2}$

1. Region 1: $\left(0 < \theta < \theta_o-\pi\right)$

$$u = \frac{m(\theta)}{\sqrt{r}} e^{ikr}$$

2. Region 2: $\left(\theta_o-\pi < \theta < 2\pi-\theta_o\right)$

$$u = U_o e^{-ikr\cos\left(\theta-\theta_o\right)} + \frac{m(\theta)}{\sqrt{r}} e^{ikr}$$

3. Regions 3 and 4: $\left(2\pi-\theta_o < \theta < \frac{3\pi}{2}\right)$

$$u = U_o e^{-ikr\cos\left(\theta-\theta_o\right)} + U_o \frac{ik\cos\theta_o+\lambda}{ik\cos\theta_o-\lambda} e^{+ikr\cos\left(\theta+\theta_o\right)} + \frac{m(\theta)}{\sqrt{r}} e^{ikr} .$$

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